Schwarzschild Atmospheric Processes: A Classical Path to the Quantum*

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We develop some classical descriptions for processes in the Schwarzschild string atmosphere. These processes suggest relationships between macroscopic and microscopic scales. The classical descriptions developed in this essay highlight the fundamental quantum nature of the Schwarzschild atmospheric processes.

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I. ATMOSPHERE AND DIFFUSION

General Relativity provides a wealth of information about the events surrounding us over a large range of scales in both space and time. We have analytic solutions to Einstein's field equations which span distances from the size of the Universe down to the size of a stellar black hole. When contemplating quantum effects at classical boundaries such as the Schwarzschild horizon, the scales become much smaller. The classical electron, a point particle, may perhaps be resolved into Planck scale strings. String bits have lengths proportional to the Planck length $\sqrt{\hbar G/c^3} = 1.6 \times 10^{-35}\ m$, and 10^{20} strings bits fit into a classical electron radius. On this smaller scale, the classical field equations can also provide insights in terms of correspondence limits. The physics of a Schwarzschild object is an arena where both large and small distance scales meet and ideas about macroscopic and microscopic correspondence can be developed and applied. In this essay we will discuss some classical and quantum aspects of the atmosphere around a Schwarzschild object.

The Schwarzschild metric, characterized by a single mass parameter, has played an important role in understanding general relativistic solutions. Vaidya [1] showed that allowing the mass parameter to be a function of retarded time created a null fluid atmosphere. Glass and Krisch [2] discovered that allowing the same parameter to be a function of the radial coordinate creates a string fluid atmosphere. In a spherically symmetric spacetime with metric $ds^2_{GK} = A(u,r)du^2 + 2dudr - r^2 d\Omega^2$, the field equations relate the mass function $m(u,r)$ to the density of the string fluid atmosphere by

$$\partial_r m = 4\pi r^2 \rho. \tag{1}$$

If one assumes that

$$\partial_u m = 4\pi D(r) r^2 \partial_r \rho \tag{2}$$

then the density obeys a diffusion equation

$$\partial_u \rho = r^{-2} \partial_r [D(r) r^2 \partial_r \rho] \tag{3}$$

with variable diffusivity $D(r)$. (In real diffusion problems, variable diffusivities are the rule rather than the exception [3].) A string fluid is a continuum description of a collection of quantum string bits. A link between the macroscopic and microscopic pictures is reflected in the density transport rule. The density diffuses in the Glass-Krisch atmosphere with $A(u,r) = 1 - 2m(u,r)/r$.

Since m describes the horizon position, the behavior of the horizon is linked to the string fluid density. Assume that the mass is diffusing in a spherically symmetric spacetime of spatial dimension δ and obeys a diffusion equation

$$\partial_u m = r^{1-\delta} \partial_r [\tilde{D}(r) r^{\delta-1} \partial_r m]. \tag{4}$$

Using Eqs.(1) and (2) and demanding consistency with Eq.(3) we find the diffusivities are equal and must have the form

$$D(r) = \tilde{D}(r) = D_0 r^{-(1+\delta)}. \tag{5}$$

If the diffusivity is constant

$$\delta = -1, \quad D(r) = D_0, \tag{6}$$

and the flowspace for mass transport then has dimension $\delta = -1$. Negative dimensions have occurred in calculations of critical behavior on random surfaces [4], [5], [6]. In this case, the negative dimension describes an internal space embedded in an external manifold of possibly different dimension. Another possible explanation of dimension δ follows from Visser's idea [7] of an hydrodynamic metric. He showed that the equation of fluid flow in a flat spacetime could be formally written in terms of a metric related to the hydrodynamic parameters of the flow. Visser's idea of an hydrodynamic flowspace is very similar to the idea of an internal space. All that the diffusion equation for $m(u,r)$ says about the flowspace is that $\sqrt{-g} = r^{\delta-1} f(\vartheta)$. The flowspace metric could be in the standard spherical form with $\delta = -1$, but it could also have any form consistent with Eq.(4).

The mass solution $m = 4\pi c(u) r^{\alpha}$ provides an interesting example of dimensional relations. Substituting into both the mass and density diffusion equations and demanding consistency, we find that the dimension of the mass flowspace is the negative of the dimension of the density space and that the diffusivity is determined:

$$\delta = -3, \quad D(r) = D_{0} r^{2}.$$

Physically the mass diffusion can be understood in terms of the position of the event horizon . An horizon whose mass depends on r creates a string fluid atmosphere. If the mass begin to change, the horizon will also begin to change its position in time. If it moves inward, then the string fluid density, also a function of position, begins to diffuse to smaller r, feeding string fluid into the regions close to the horizon. If the horizon is growing then the string fluid diffuses outward to larger r.

II. A CLASSICAL ANALOGY

The string fluid is serving as a dissipative atmosphere around a Schwarzschild horizon. The central Schwarzschild object draws upon the string fluid, trying to maintain the horizon position. This is somewhat analogous to the behavior of an inductive circuit with the string fluid attempting to counter the changing horizon position as an inductor tries to counter changes in magnetic flux. This analogy can be carried further by noting that in a lossy transmission line, the current and voltage obey the equations

$$\begin{aligned} L I_{,t} + R I &= -V_{,w} \\ C V_{,t} + G V &= -I_{,w} \ , \end{aligned} \tag{7}$$

where the circuit parameters, L, R, G, and C, are all per unit length. These first order equations are equivalent to a second order telegrapher's wave equation for I or V.

If we require that the transmission line have $R = C = 0$ then these equations are formally identical to equations (1) and (2) with $I \to m$, $V \to \rho$, $t \to u$, $w = 1/r$, and where the inductance and leakage inductance densities are given by

$$L = 1/4\pi D(w) \tag{8}$$
$$G = 4\pi/w^4.$$

A similar analogy with only resistance and capacitance has $G \rightarrow R$, $L \rightarrow C$, and $V \rightarrow m$. It is not surprising that the atmosphere can be modeled as a transmission line since the string fluid and null fluid each serve as a transmission medium for the other. An interesting aspect of this classical analogy is the "lossiness" of the atmosphere, either through resistance or leakage inductance.

III. DIFFUSION AND DISSIPATION

The description of a string atmosphere diffusing inward as null radiation moves outward classically models underlying quantum processes. The dissipation in the electrical analogy models quantum friction. Quantum dissipation is usually the result of coupling between a system and a complex environment [8]. The coupling causes information loss. There have been several recent suggestions about the origin of quantum friction in stringy systems. Ellis et al [9] have suggested that when light particles scatter from D-branes, neglecting the D-brane recoil results in information loss. They also find, using Renormalization Group arguments, that the D-brane wave function evolves diffusively. A related suggestion by the same authors [10] considers the interaction of light particles with an environment of spacetime foam. In this calculation, quantum friction is due to couplings with unobserved massive string states. Both examples explain quantum friction as due to information loss in a scattering process. Diffusion is, of course, intrinsically dissipative since in a diffusive process one may only predict and not retrieve. A possible origin for the dissipation in the string atmosphere is through interactions between the null radiation and the string fluid. This can be examined in a simple model calculation. Consider a static model with $m = m(r)$ and a string fluid density $\rho = \rho_0 + k_1/r$. From (1) the string mass is $m(r) = m_0 + (4\pi/3)r^3\rho_0 + 2\pi k_1 r^2$. If the mass is allowed to be a function of retarded time and $D(r) = D_0$, this particular density will not change but the mass is now

$m(u,r) = m_0 + (4\pi/3)r^3\rho_0 + 2\pi k_1(r^2 - 2D_0 u)$. We can identify the first three terms as the string mass and the last term can be interpreted as the net flux through an $r = constant$ spacelike 3-surface [2]. The net flux is due to the outward moving radiation and the inward diffusing string bits. The constant diffusivity parametrizes the net flux. A diffusivity is a measure of the resistance offered to the diffusing medium by the surrounding environment [11] so this term could represent the interaction between the null fluid created by $\partial_u m$ and the string fluid. There are several other possible sources of friction. The complete extension of the Vaidya metric generates fluid with both radial and transverse stress. The transverse stress can be attributed to dust [2] or to a non-zero magnetic contribution to the string bivector [12]. String-dust scattering could be a source of information loss. Another possible loss mechanism is the snapping of tidally stretched string bits [13], the snap and loss of potential energy, sending a disturbance through the surrounding environment.

IV. SCALE AND OTHER TRANSPORT PROCESSES

We have seen in the previous section that the diffusion can provide significant insights into the behavior of the horizon. We assumed a diffusion equation for the atmospheric string density and found the resulting diffusion equation for the mass; for $D(r) = D_0$ each scale with the Boltzmann scaling variable η [15], where $\eta^2 = \frac{r^2}{4D_0 u}$. This scaling variable is traditionally associated with simple diffusive mass transport. Another source of similarity behavior and scaling variables in the Schwarzschild system is the metric itself [16]. Because the atmospheric string fluid lives on a 2-dimensional world sheet in the (u, r) plane, the similarity behavior of the matter surfaces is of interest. Starting with an assumed scaling behavior, we try to develop an associated mass transport rule.

The Glass-Krisch metric can be written in terms of unit vectors as

$$g_{ab}^{GK} = \hat{v}_a\hat{v}_b - \hat{r}_a\hat{r}_b - \hat{\vartheta}_a\hat{\vartheta}_b - \hat{\varphi}_a\hat{\varphi}_b$$

(see [2] for details). The (u, r) world sheet is spanned by unit vectors $\hat{v}_a$ and $\hat{r}_a$ and is scaled by

$$\mathcal{L}_\xi(\hat{v}_a\hat{v}_b - \hat{r}_a\hat{r}_b) = 2\mu(\hat{v}_a\hat{v}_b - \hat{r}_a\hat{r}_b). \tag{9}$$

Similarly, the orthogonal (ϑ, φ) 2-surfaces are spanned by $\hat{\vartheta}_a$ and $\hat{\varphi}_a$ and are scaled by

$$\mathcal{L}_\xi(\hat{\vartheta}_a\hat{\vartheta}_b + \hat{\varphi}_a\hat{\varphi}_b) = 2\nu(\hat{\vartheta}_a\hat{\vartheta}_b + \hat{\varphi}_a\hat{\varphi}_b). \tag{10}$$

The similarity vector which preserves the distinct (u, r) 2-surfaces of the matter distribution is

$$\xi^a\partial_a = [\nu u_{_0} + (2\mu - \nu)u)]\partial_u + \nu r\partial_r.$$

The scale symmetry of Eqs.(9) and (10) requires a first order differential constraint on $g_{uu} = A = 1 - 2m(u, r)/r$.

$$(u_{_0} + \kappa u)A_{,u} + rA_{,r} = (1 - \kappa)A. \tag{11}$$

where $\kappa := 2\mu/\nu - 1$. The assumption of a similarity transform on the matter 2-surfaces has imposed a mass transport rule. This is recognizable as a first order form of the telegrapher's equation. A lossy transmission line analogy is possible here ([11] p. 219). In terms of coordinates $t = \kappa^{-1}\ln(u_{_0} + \kappa u)$ and $q = \ln(r)$, the second order form of Eq.(11) is

$$A_{,tt} - A_{,qq} + (\kappa - 1)(A_{,q} - A_{,t}) = 0$$

where the presence of $A_{,t}$ indicates damping of wave motion.

When $\kappa = 1$ and $(\mu, \nu) = (1, 1)$ then A obeys a simple wave equation on the flat tangents to the string 2-space. There is no dissipation. Choosing $\kappa = 1$ makes the map homothetic on the entire spacetime, $\mathcal{L}_\xi g_{ab} = 2g_{ab}$. Since density and mass are related by Eq.(1), we obtain a second order equation for the density from Eq.(11)

$$\rho_{,tt} - \rho_{,qq} + (\kappa - 1)\rho_{,t} = (\kappa + 3)\rho_{,q} + 2(\kappa + 1)\rho$$

and we again find the dissipative term vanishing for a homothetic map.

Other parameter values will include diffusive effects. For example $(\mu, \nu) = (1/2, 1)$, $\kappa = 0$, preserves the scale of the string 2-surfaces while acting homothetically on the orthogonal

space. For these parameters one can show from Eq.(11) that an associated second order equation is (with a more well known form of the telegrapher's equation)

$$A_{,uu} - 3A_{,u}/u_0 - (r/u_0)^2\nabla^2 A = -2A/u_0^2. \tag{12}$$

V. CONCLUSION

To summarize, we have seen that the string fluid atmosphere around a Schwarzschild black hole can have profound effects on the mass transport which drives the horizon position. Common elements of the mass transport considered are the existence of a scaling variable and analogies to classical dissipative systems. This is an especially interesting result to hold across several models since t'Hooft [17] has suggested the necessity for dissipation of information as an ingredient of a theory of quantum gravity. The absence of dissipation in the mass transport associated with a homothetic map is also very interesting, given his suggestion. The presence of both the diffusion equation and the telegrapher's equation [18] is also highly suggestive of the underlying quantum nature of the atmospheric processes since both of these equations have been used to link macroscopic and microscopic physics. The classical descriptions developed in this essay highlight the fundamental quantum nature of the Schwarzschild atmosphere.

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